Injection Locking Efficiency of Two Independent Lasers

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ABSTRACT

Bidirectional (mutual) injection locking was demonstrated with solid-state lasers, producing significant improvements over traditional single-direction injection locking. Each laser element shares part of its output with other elements in bidirectional locking, distinct from single-direction (traditional) injection locking where one master laser provides the locking signal for a number of slaves. In a phase-locked array, the individual laser outputs add coherently, and the brightness of the entire array scales with the square of the number of elements, as if the active material diameter were increasing. Benefits of bidirectional locking, when compared to traditional injection locking, include reduced laser threshold, better output beam quality, and improved scaling capability. Experiments using two Nd:YVO₄ lasers confirmed that mutual injection locking effects began with 0.03% coupling between lasers and full-phase locking for coupling exceeding 0.5%. The 0.5% requirement for full phase-locking limits traditional injection-locked arrays to fewer than 100 elements, while mutually injection-locked arrays have no such limit. Mutual injection locking of an array of lasers can lead to a new architecture for high-power laser systems.

1. INTRODUCTION

In the quest for ever higher power in solid-state lasers—with an immediate target of 100 kW average power¹⁻⁴—three technologies have emerged as the best candidates. These three are disc laser stacks, such as the Heat Capacity Laser (HCL) being developed at Lawrence Livermore Laboratories^{3.5}; slab and other master oscillator/power amplifier (MOPA) lasers, under development at Northrop Grumman^{6.7} and Coherent Technologies; and laser arrays, the subject of this paper. An HCL can produce a high output, but with a low duty cycle. This requires that a collection of HCLs be multiplexed for continuous operation. MOPA systems have been demonstrated at over 5 kW of average power, running continuously⁸, but higher power requires more stages and trickier alignments. An optimum 100 kW laser, for example, would require some 20 times as much space, cooling, pump power, etc., as the current 5 kW system. Additionally, such a system is limited by crystal growth technology. With the largest commercial Nd:YAG laser amplifiers, for example, the beam intensity in such a laser is sufficient to cause self-focusing and other nonlinear phenomena in the final few stages, reducing efficiency. What is needed for a usable 100 kW laser is a system that is efficient (reducing the size of the power supply and cooling mechanism), not overly large (it must fit in whatever space is allotted), and easy to cool (to avoid severe thermal effects). A laser array meets all these requirements, and produces a high-quality beam as long as the oscillators are phase-locked together^{9,10}.

2. VALUE OF INJECTION LOCKING

One method of phase-locking several oscillators is to extract a small portion of the oscillating beam in one laser cavity and inject it into another cavity (Fig. 1). The most common method is to inject the oscillating signal from one laser (the master) into the cavities of several others (the slaves). Injection-locked lasers have many advantages over monolithic laser systems, as described in the following sections.

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Fig. 1. A simple injection locking system, in which a small part of the power oscillating in the master cavity is shared with each of n slave oscillators.

2.1. Power scaling

The greatest advantage of injection locking is the ease of scaling to high power. The overall output power of the laser array is just the sum of the individual element powers. To increase power, it is simply necessary to increase the number of elements in the array. For example, if each of 200 elements has an output of 500 W, the total array output will be 100 kW. Furthermore, the array elements are added parallel to each other, not in series, overcoming the difficulties of MOPA systems – unacceptably high intensity in the amplifiers and extreme alignment sensitivity.

2.2. Beam quality and coherence

If the beams are merely combined, in a beam combiner or by pointing at the same object, they add incoherently. The total power is the sum of the individual powers, but the combined beam expands at a divergence angle approximately equal to that of the individual elements. The brightness of the beam, or maximum intensity in the far field, increases linearly with the number of elements (assuming each element produces the same output). Locking the wavelength and phase of the beams improves this significantly. Phase-locked lasers add coherently¹¹, increasing the brightness by the square of the number of elements (Fig. 2). This occurs because the combined laser beam's effective divergence angle is proportional to the inverse of the number of elements, even as the combined power is proportional to that number. If each 500 W laser rod in our example is capable of being focused to an intensity of 10 kW/cm² at a given distance, even though the total power only increases to 100 kW, the intensity of a phase-locked array at the given distance can reach 400 MW/cm². This is in sharp contrast to incoherent addition, which only increases the intensity to 2 MW/cm².



Fig. 2. Simulation of near-field (a) and far-field (b) brightness of a four-laser array, showing a factor of 16 increase in brightness as the four individual beams add coherently.

2.3. Bandwidth, threshold, and lasing wavelength

Injection-locked lasers exhibit bandwidth reduction compared to the same lasers when not injection-locked. This occurs if the injected laser is narrowband compared to the free-running laser^{12,13}, but also when the injected signal is broadband¹⁴. The laser wavelength can be "pulled" to the wavelength of the injected signal¹⁵, enabling a small tuning range if the injected signal can vary. If the lasers are connected through mutual injection locking, the threshold of each laser is reduced compared to the free-running cavity, and the power extraction efficiency is increased^{10,16}.

3. INJECTION LOCKING THEORY

Injection locking has been known and used for a long time¹⁷. One of the best derivations of laser injection locking theory was produced by Siegman⁹, and that derivation is followed here. First, consider a laser oscillator operating below threshold. Acting as an amplifier, its single-pass amplitude gain $g(\lambda)$, at wavelength λ , may be written

$$g(\lambda) = \frac{1 - R_1 R_2}{1 - G_{\text{RT}}(\lambda) e^{-i\phi(\lambda)}} , \qquad (1)$$

where R_1 and R_2 are the reflectivities of the oscillator mirrors, $G_{RT}(\lambda)$ is the total round-trip gain at wavelength λ , and $\phi(\lambda)$ is the round-trip phase. Operation below threshold is chosen for two reasons: (1) the injected signal in an injection-locked laser is usually below the lasing threshold, and (2) above threshold the round-trip gain is unity and the round-trip phase is a multiple of 2π , so $g(\lambda)$ Eq. (1) is infinite. Now assume the laser is oscillating at wavelength λ_0 . Then the amplification of a signal at wavelength λ_1 λ_0 is

$$g(\lambda_1) = \frac{1 - R_1 R_2}{1 - G_{RT}(\lambda_1) e^{-i\phi(\lambda_1)}} \approx \frac{1 - R_1 R_2}{i 4\pi L} \frac{\lambda_1 \lambda_0}{\lambda_1 - \lambda_0}$$
(2)

for $\lambda_1 \ \lambda_0$, so that $G_{RT}(\lambda_1)$ 1. In Eq. (2), L is the optical path length (single-pass) of the resonator. The amplified intensity is thus

$$\mathbf{I}(\lambda_1) = \left| g(\lambda_1) \right|^2 \mathbf{I}_1 \approx \left(\frac{1 - \mathbf{R}_1 \mathbf{R}_2}{4\pi \mathbf{L}} \right)^2 \left(\frac{\lambda_1 \lambda_0}{\lambda_1 - \lambda_0} \right)^2 \mathbf{I}_1 \quad , \tag{3}$$

where I_1 is the intensity of the injected signal. As $\lambda_1 \rightarrow \lambda_0$, the amplified injected signal can exceed the intensity of the freerunning signal. In a homogenously broadened medium, at that level of injected intensity the laser will move its oscillation wavelength from λ_0 to λ_1 , and injection locking will be complete. The oscillator signal will match the wavelength of the injected signal, although the phase may be different. Locking will occur if the wavelength is within the range

$$\lambda_0 \left[1 - \frac{\lambda_0}{4\pi L} \left(1 - R_1 R_2 \right) \sqrt{\frac{I_1}{I_0}} \right] < \lambda_1 < \lambda_0 \left[1 + \frac{\lambda_0}{4\pi L} \left(1 - R_1 R_2 \right) \sqrt{\frac{I_1}{I_0}} \right]$$
(4)

Eq. (4) can be rewritten to solve for the needed injection intensity:

$$\frac{I_1}{I_0} > \left(\frac{4\pi}{1 - R_1 R_2} \frac{L}{\lambda_0} \frac{\lambda_0 - \lambda_1}{\lambda_0}\right)^2$$
(5)

(using the relationship $\lambda_1 \ \lambda_0$). For example, if the laser is Nd:YAG, the optical length of the cavity is 10 cm, the mirror reflectivity product is 90%, and the wavelength shift between λ_1 and λ_0 corresponds to 5 MHz, the injected intensity only needs to be 3% as strong as the oscillating intensity. On the other hand, if the wavelength shift corresponds to 7.5 MHz, the injected intensity must be 10% of the oscillating intensity. The relationship between needed intensity and signal offset for this case is shown in Fig. 3.



Fig. 3. Injected intensity required for locking as a function of difference between the injected and free-running wavelengths. The gray areas indicate successful injection locking.

Examination of Fig. 3 demonstrates that injection locking requires either a very strong injected signal or an injection laser whose wavelength is very close to that of the free-running oscillator to be locked. Since the purpose of injection locking is usually to lock a strong laser to a weaker signal, the injected signal in this case must be within ~10 MHz of the free-running signal. It is common for solid-state lasers to have free-running wavelengths well within this range, ensuring ease of injection locking, but some form of frequency tuning or adjustment may be necessary to ensure it.

4. BIDIRECTIONAL VS. MASTER-SLAVE

The sections above discussed injection locking solely from the standpoint of unidirectional locking, where one laser is the master and the others (those receiving the injection) are its slaves. There is another way—a small part of the oscillating energy in each of the lasers can be injected into the others. This can lead to unique benefits.

4.1. Theoretical difference

There is little theoretical difference between unidirectional and bidirectional injection locking. In most unidirectional cases, referring again to Fig. 2, all involved lasers (the master and all the slaves) are oscillating before injection occurs; in mutual injection locking (Fig. 4), all the lasers originally lock to the free-running oscillation of the first laser to reach threshold.



Fig. 4. Mutual injection locking, in which two lasers share portions of each other's oscillating field.

However, there are some theoretical differences between the two types of injection locking. Three important differences are the output power of each laser, the phase of the injection-locked laser with respect to the injected signal, and the laser output wavelength. For example, the output intensity of a single slave injection-locked laser can be shown to be⁹

$$I(\lambda_1) = \frac{I_{\text{sat}}}{I_{\text{sat}} - \sqrt{I_1 I_0} \cos \phi(\lambda_1)} I_0 \quad , \tag{6}$$

where I_{sat} is the saturation intensity of the laser, I_1 is the intensity of the injected signal at wavelength λ_1 , I_0 is the amplitude of the free-running signal at λ_0 , and ϕ is the laser output phase,

$$\phi(\lambda_1) = \phi_1 + \sin^{-1} \left(\frac{\lambda_1 - \lambda_0}{\lambda_1 \lambda_0} \lambda_m \right) .$$
⁽⁷⁾

In Eq. (7), ϕ_1 is the phase of the injected signal (which may be set to 0 by referring all signals to the injected signal) and

$$\lambda_{\rm m} \equiv \frac{\lambda_0^2}{4\pi L} \left(1 - R_1 R_2 \right) \sqrt{\frac{I_1}{I_0}} \tag{8}$$

is the maximum detuning for injection locking. Since $\lambda_m \ll \lambda_0, \lambda_1$, Eqs. (6, 7) may be rewritten

$$\mathbf{I}(\lambda_1) \approx \frac{\mathbf{I}_{\text{sat}}}{\mathbf{I}_{\text{sat}} - \sqrt{\mathbf{I}_1 \mathbf{I}_0}} \mathbf{I}_0 = \frac{\mathbf{I}_{\text{sat}}}{\mathbf{I}_{\text{sat}} / \mathbf{I}_0 - \sqrt{\mathbf{I}_1} / \mathbf{I}_0} , \qquad (9)$$

where I_{sat} is the laser material saturation intensity, and

$$\phi(\lambda_1) \approx \frac{\lambda_1 - \lambda_0}{\lambda_1 \lambda_0} \lambda_m \approx \frac{\lambda_1 - \lambda_0}{4\pi L} \left(1 - R_1 R_2 \right) \sqrt{\frac{I_1}{I_0}} \quad . \tag{10}$$

Eq. (10) demonstrates that the phase difference between the injected laser and the output signal is very small, since it is the product of two small ratios: $\lambda_1 - \lambda_0 \ll L$ and $I_1 \ll I_0$. Eq. (9) shows an interesting effect: The output from the amplified injected laser is actually larger than the sum of the injected and free-oscillating signals. The wavelength of the output is essentially λ_1 , the wavelength of the injected signal (some very minor wavelength pulling may occur because of cavity length).

Mutual injection locking leads to some slight modifications to the previous equations. For example, the exact wavelength of the two lasers shifts to a wavelength between the two free-running wavelengths. In the case where both cavities are oscillating and the injection is symmetric (that is, the same percentage of oscillating intensity from Cavity 1 is injected into Cavity 2 as vice versa), the ratio of the intensities of the two lasers is

$$\frac{I_2}{I_1} \approx \frac{g_2^2(\lambda_0)}{g_1^2(\lambda_0)} \frac{1 - R_{1,1}R_{2,1}}{1 - R_{1,2}R_{2,2}} , \qquad (11)$$

where I_n is the intensity oscillating in Cavity n (n $\in \{1, 2\}$), g_n is the single-pass gain described by Eq. (2),

$$\lambda_0 \equiv \frac{\lambda_1 + \lambda_2}{2} \tag{12}$$

is the approximate combined laser wavelength, and $R_{m,n}$ is the reflectivity of Mirror m in Cavity n. Defining the injection percentage as α , we find the injection wavelength range to be

$$\lambda_{2} \left[1 - \frac{\lambda_{2}}{4\pi L} \frac{g_{2}(\lambda_{0})}{g_{1}(\lambda_{0})} \sqrt{\alpha \left(1 - R_{\text{tot},1} \right) \left(1 - R_{\text{tot},2} \right)} \right] < \lambda_{1} < \lambda_{2} \left[1 + \frac{\lambda_{2}}{4\pi L} \frac{g_{2}(\lambda_{0})}{g_{1}(\lambda_{0})} \sqrt{\alpha \left(1 - R_{\text{tot},1} \right) \left(1 - R_{\text{tot},2} \right)} \right]$$
(13)

The individual mirror reflectivities have been replaced by a total reflectivity, $R_{tot,n} = R_{1,n}R_{2,n}$. In other words, a mutually injection locked laser has bandwidth and output properties that can be directly determined by adjustable parameters of the laser system. The bandwidth, for example, depends on the ratio of the single-pass gain in the two lasers, the geometric mean of their output coupling, and the injection percentage, while the ratio of the output powers depends on the ratio of the power gain in the two cavities and the ratio of their output coupling. Additionally, the frequency-pulling effects result in both lasers' outputs being at a wavelength that is between the free-running wavelengths of the independent oscillators. Thus, the injection locking bandwidth is greater and the phase difference between the outputs is less than in the case of master-slave injection locking.

4.2. Practical difference

The practical differences between mutual injection locking and master-slave injection locking are greater than would be assumed from the theoretical differences. While the theory shows differences in output power and wavelength, the most obvious practical differences are reduced threshold and improved mode structure^{10,14}. These are both caused by the same change from free-running oscillators: the addition of an extra cavity that shares the two laser elements (Fig. 4).

The extra cavity, referred to in Fig. 4 as Cavity 3, is longer than Cavity 1 or Cavity 2. It has much higher loss, the product of two output couplers and two coupling mirrors. It also has two gain media, in this case the two laser rods. In some cases, Cavity 3 mirror reflectivities may be large enough that this cavity begins to oscillate before the other two. In any case, its length and loss generate a lower-order mode than either Cavity 1 or Cavity 2 would be likely to have on its own. This layout could be considered as Cavity 3 injecting into Cavities 1 and 2.

The longer cavity tends to generate a smaller mode, which concentrates the gain of Cavities 1 and 2 near the center of their rods. This effect, together with the tendency toward lower-order modes described above, reduces the lasing threshold of both Cavity 1 and Cavity 2. Thus, the mutually injection-locked laser pair has lower threshold, higher output, and better mode structure than a single rod cavity without injection locking.

5. BIDIRECTIONAL INJECTION LOCKING EXPERIMENT

5.1. Layout and calculations

The injection locking experiment comprised two cavities, each with a Nd:YVO₄ laser crystal end-pumped at 532 nm (Fig. 5). At 532 nm, the Nd:YVO₄ absorptivity is nearly a factor of 10 higher for c-polarized light than for a-polarized, so the pump laser polarization was aligned with the crystal optical axis. The emission at 1064 nm, however, was a-polarized.



Fig. 5. Bidirectional injection locking experiment.

The pump source for this experiment provided up to 8 W CW at 532 nm. This beam was chopped, then passed through a 50/50 beamsplitter to pump both cavities. The beam was focused through the HR of each cavity, with the focal spot inside the crystal. Between the forward face of the crystal and the OC, each cavity had a pickoff mirror; the mirrors were aligned to reflect each beam exactly along the path of the other cavity beam. Between the two pickoff mirrors was a rotatable polarizer, used to vary the amount injected from each cavity into the other. The measured reflectivity of the HRs was 99.2% at 1.064 μ m and normal incidence, that of the OCs was 95.8%, and the pickoff mirror reflectivity was 16.4% for 45° incidence (so the total coupling between cavities could be varied from 0 to 2.69%). Raytracing demonstrated that the pump beam diameter inside the top cavity in Fig. 5 (referred to as Cavity 1) was 950 μ m at the entrance to the laser crystal, while the pump beam diameter for each cavity has been calculated to be 365 μ m.

5.2. Independent operation

Each cavity was operated with the pickoff mirrors removed to avoid any injection. Each was tested with output couplers of differing reflectivities, enabling a determination of the intracavity losses. The results appear in Fig. 6.



Fig. 6. Threshold vs. Loss Plot for Independent Cavities.

The threshold was lower in Cavity 2 than in Cavity 1, expected as the pump intensity was higher. The threshold power was measured directly from the laser, ignoring input losses, chopper duty cycle, and pump light transmission. The intracavity losses of both cavities are approximately equal, calculated from this graph at 3% per round trip. The mode quality of Cavity 2 was fair, while that of Cavity 1 was poor (Fig. 7).



Fig. 7. Independent operation of (a) Cavity 1 and (b) Cavity 2 in the bidirectional injection locking experiment. The output of Cavity 1 is highly multimode, while that of Cavity 2 represents lower-order modes.

5.3. Transitional range

The lasers were operated again with the coupling in place, but with the 10,000:1 polarizer set to block injection. The cavities operated exactly as they did when the pickoff mirrors were removed, except that the threshold was higher due to the insertion loss of the pickoff mirrors (1.9 W laser power instead of 1.1 W for Cavity 2, 6.3 W instead of 2.6 W for Cavity 1). The first effects of injection locking, a lowering of the lasing threshold for both cavities, were seen when mutual injection was increased to 0.03%. As coupling was increased, the threshold dropped even more (see Table 1), until, at 0.5% mutual injection the threshold was at its lowest, even with the maximum coupling available, 2.7%. The mode quality improved as well, as coupling was increased past 0.03%, especially in Cavity 1. Finally, at 0.5% coupling, the modes of both lasers were good, and the lasers could be considered fully injection-locked. This corresponds to a free-running oscillation frequency difference of ~10 MHz, twice the frequency separation indicated by this measurement in a unidirectional injection locking laser.

Threshold	Cavity 1	Cavity 2
Coupling mirrors removed	2.6 W	1.1 W
Mirrors in place, coupling $= 0$	6.3 W	1.9 W
Coupling = 0.03%	2.2 W	1.8 W
Coupling = 0.1%	1.9 W	1.5 W
Coupling $= 0.5\%$	1.8 W	1.4 W
Coupling $= 2.7\%$	1.8 W	1.4 W

Table 1. Comparison of laser threshold from Cavities 1 and 2 to demonstrate injection locking. Threshold power is measured at the pump laser. For this table, threshold is defined as >50 μW output from the cavity.

The data in Table 1 demonstrate that injection coupling is a good method of lowering the threshold of lasers. It is also clear that the majority of the threshold-lowering effect appears with less coupling than that needed to phase-lock the lasers (0.5%). It is interesting to note that even 0.03% coupling is enough to reduce lasing threshold in Cavity 1 to below its level when operated independently, even with 4.2× as much loss. In fact, theory shows that, as long as Cavity 2 is oscillating and the coupling is sufficient for full injection locking, both cavities oscillate. This was not observed in our experiment because, at the lower pump levels, the laser output of Cavity 1 was <50 μ W (our definition for threshold).

5.4. Full locking

When the two laser cavities are fully injection locked, their phase is locked to some small (and constant) difference. This can be seen from Fig. 8, which shows the results of mixing the two beams together. As injection percentage increases, laser threshold drops (Table 1), and beam quality increases (compare Figs. 7(a) and 8(a)). At approximately 0.5% injection in this experiment, the threshold and beam quality effects have saturated, and the two laser outputs are locked in phase and wavelength.



Fig. 8. Combined output of two laser beams in mutual injection locking experiment. With 0.45% injection (a), threshold is reduced and beam quality is improved over unlocked beams. With 0.5% injection (b), the outputs are phase locked as well, as shown by the horizontal interferometric fringes.

6. CONCLUSIONS

Mutual injection locking of lasers is a promising technology for the next generation of high-power laser arrays. This technique reduces laser threshold in the associated cavities, increases the lockable difference between the free-running laser wavelengths of the lasers, and reduces the phase difference between the locked oscillators. An experiment demonstrated that the laser thresholds of two independent Nd:YVO₄ laser cavities, pumped by a single 532-nm laser, were reduced up to a factor of 3.5 with significant mode quality improvement when the two cavities were mutually injection locked. In this

experiment, 0.5% mutual injection locking was required to fully phase-lock the two cavities, as would be expected in a master-slave injection-locked laser where the injected signal was separated from the free-running oscillator signal by \sim 5 MHz. The relationship between frequency separation of the injected and free-running signals to the required injection power puts severe limits on the capabilities of master-slave injection-locked lasers. These limits are overcome by mutual injection locking.

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