# Mutual Injection Locking: A New Architecture for High-Power Solid-State Laser Arrays

Russell M. Kurtz, *Senior Member, IEEE*, Ranjit D. Pradhan, Nay Tun, Tin M. Aye, Gajendra D. Savant, Tomasz P. Jannson, and Larry G. DeShazer, *Fellow, IEEE* 

Abstract-In this paper, bidirectional (mutual) injection locking is demonstrated with solid-state lasers, producing significant improvements over traditional single-direction injection locking. Each laser element shares part of its output with other elements in bidirectional locking, distinct from single-direction (traditional) injection locking where one master laser provides the locking signal for a number of slaves. In a phase-locked array, the individual laser outputs add coherently, and the brightness of the entire array scales with the square of the number of elements, as if the active material diameter were increasing. Benefits of bidirectional locking, when compared to traditional injection locking, include reduced laser threshold, better output beam quality, and improved scaling capability. Experiments using two Nd:YVO4 lasers confirmed that mutual injection locking reduced lasing threshold by a factor of at least two and increased the output beam quality significantly. The injection-locking effects began with 0.03% coupling between lasers and full-phase locking for coupling exceeding 0.5%. The 0.5% requirement for full-phase locking is significantly lower than the requirement for traditional injection locking. The large coupling requirement limits traditional injection-locked arrays to fewer than 20 elements, whereas mutually injection-locked arrays have no such limit. Mutual injection locking of an array of lasers can lead to a new architecture for high-power laser systems.

Index Terms-Injection-locked oscillators, laser arrays, solid lasers.

## I. INTRODUCTION

N THE quest for ever higher power in solid-state lasers, three technologies have emerged as the best candidates: disc laser stacks, such as the heat-capacity laser (HCL) being developed at Lawrence Livermore Laboratories [1], [2]; slab and other master-oscillator/power-amplifier (MOPA) lasers, under development at Northrop Grumman [3], [4] and Coherent Technologies; and laser arrays, the subject of this paper. An HCL can produce a high output, but with a low duty cycle. Therefore, continuous operation requires that a collection of HCLs be multiplexed. MOPA systems have been demonstrated at over 5 kW of average power at a 20% duty cycle [3], but higher power requires more stages and trickier alignments. An optimum 100kW laser, for example, would require some 20 times as much space, cooling, pump power, etc. as the current 5-kW system. Additionally, such a system is limited by crystal growth technology. With the largest commercial Nd:YAG laser amplifiers, for example, the beam intensity is sufficient to cause self-focusing

Manuscript received August 31, 2004; revised April 14, 2005.

R. M. Kurtz, R. D. Pradhan, N. Tun, T. M. Aye, G. D. Savant, and T. P. Jannson are with Physical Optics Corporation (POC), Torrance, CA 90501 USA.

L. G. DeShazer is with the Center for Applied Competitive Technologies,

Irvine Valley College, Irvine, CA 92618. Digital Object Identifier 10.1109/JSTQE.2005.850240



Fig. 1. In a traditional injection locking system, a master oscillator injects part of its output into at least one slave oscillator.

and other nonlinear phenomena in the final few stages, reducing efficiency.

What is needed for a usable high-power laser is a system that is efficient (reducing the size of the power supply and cooling mechanism), not overly large (it must fit in whatever space is allotted), and easy to cool (to avoid severe thermal effects). A laser array meets all these requirements and produces a highquality beam as long as each laser's frequency is locked to the others by some method, such as injection locking [5], [6].

## II. VALUE OF INJECTION LOCKING

Injection locking enables adjacent laser oscillators to operate at the same frequency and with constant phase difference. It is accomplished by injecting a portion of one laser output or oscillation into the oscillating cavity of at least one other laser (Fig. 1). Ordinarily, the high-quality signal from one laser (the master) is split and injected into several others (the slaves). This approach has many advantages over single-oscillator and MOPA systems, as described in the following paragraphs.

### A. Power Scaling

The greatest advantage of injection locking is the ease of scaling to high power. The overall output power of the laser array is the sum of the individual element powers. To increase power, it is simply necessary to increase the number of elements in the array. For example, if each of 200 elements has an output of 500 W, the total array output is 100 kW. Furthermore, the array elements are added parallel to each other, not in series, overcoming

#### 1077-260X/\$20.00 © 2005 IEEE

Copyright © 2005 IEEE. This paper is available as an electronic reprint with permission of IEEE. One print or electronic copy may be made for personal use only. Systematic or multiple reproduction, distribution to multiple locations via electronic or other means, duplication of any material in this paper for a fee or for commercial purposes, or modification of the content of the paper are prohibited.



Fig. 2. (a) In the near field, a four-element phase-locked array acts as four independent oscillators, as does an incoherently combined array in the far field. (b) The phase-locked array output, however, combines into a single narrow high-quality beam. All beams were modeled as lowest order Gaussian and propagated with the Fraunhofer propagation equation.

the difficulties of MOPA systems: unacceptably high intensity in the amplifiers and extreme alignment sensitivity.

#### B. Beam Quality and Coherence

If the beams are merely combined in a beam combiner or by pointing at the same object, they add incoherently. The total power is the sum of the individual powers, and each beam expands at its own rate so that the combined beam expands at a rate approximately equal to the average of the individual elements' divergence angle. The brightness of the beam, or maximum intensity in the far field, increases linearly with the number of elements (assuming all elements produce the same output). Locking the wavelength and phase of the beams improves this significantly. Phase-locked lasers add coherently [7], increasing the brightness by the square of the number of elements (Fig. 2). This effect occurs because the combined laser beam's effective divergence angle is proportional to the inverse of the number of elements, even as the combined power is proportional to that number. If each 500-W laser rod in our example is capable of being focused to an intensity of 10 kW/cm<sup>2</sup> at a given distance, even though the total power only increases to 100 kW, the intensity of a phase-locked array at the given distance can reach 400 MW/cm<sup>2</sup>. This is in sharp contrast to incoherent addition, which only increases the intensity to 2 MW/cm<sup>2</sup> (lower by a factor equal to the number of elements).

 TABLE I

 Parameters Used in Injection-Locking Calculations

Symbol	Description	Formula
р	Optical length of one round trip	
	("perimeter"), cm	$\int n  dl$
$R_H$	Combined reflectivity of all mirrors except output coupler, %	$R_1R_2R_n$
$R_O$	Reflectivity of output coupler, %	$R_0$
η	Injection efficiency, %	I <sub>1</sub> /I <sub>inj</sub>
α	Round-trip loss parameter, cm <sup>-1</sup>	-ln(total loss)/p
g	Round-trip gain parameter, cm <sup>-1</sup>	ln( <i>total gain</i> )/p
$\gamma_r (1/\tau_{rt})$	Inverse of cavity round-trip time, s <sup>-1</sup>	c/p
$\gamma_c (1/\tau_c)$	Decay rate of photons in the cavity, s <sup>-1</sup>	$-\alpha \gamma_r \ln(R_H R_O)$
$\gamma_e (1/\tau_e)$	Rate of output coupling, s <sup>-1</sup>	$-\gamma_r \ln(R_O)$
Ym	Gain rate, s <sup>-1</sup>	g p Yr

#### C. Bandwidth, Threshold, and Lasing Wavelength

Injection-locked lasers exhibit bandwidth reduction compared to the same lasers when not injection-locked. This is most noticeable if the injected laser is narrow-band compared to the free-running laser [8], [9], but even occurs when the injected signal is broadband [10]. The laser wavelength is locked to the wavelength of the injected signal [11]. If the lasers are connected through mutual injection locking, the threshold of each laser is reduced compared to the free-running cavity, and the power extraction efficiency is increased [12], [13].

# **III. UNIDIRECTIONAL INJECTION LOCKING THEORY**

## A. Locking Range

Injection locking of lasers was first accomplished in 1966. Several methods have been used to describe the result of locking two lasers together with light injection [9], [14]–[17], with varying success. The most general analysis starts with the slowly varying envelope approximation (SVEA) of Maxwell's equations, which is valid for virtually any laser. If an oscillator with a homogeneously broadened gain medium and natural radian frequency  $\omega_0$  is driven by a signal with radian frequency  $\omega_1$ , amplitude  $E_1$ , and phase difference  $\varphi$  compared to the natural signal, the differential equation describing the total field amplitude *E* is

$$\frac{dE}{dt} + \frac{\gamma_{\rm c} - \gamma_{\rm m}}{2} E = \gamma_{\rm r} E_1 \cos \varphi \tag{1}$$

whereas the equation describing the phase is

$$\frac{d\varphi}{dt} + (\omega_1 - \omega_0) = \gamma_r \frac{E_1}{E} \sin \varphi.$$
 (2)

The variables and constants in these equations are shown in Table I. In the most general case, all the terms in (1) and (2) may vary with time. In the case of injection locking, however, several approximations can be made. The rates and frequencies can be assumed to be constant over time, the injected signal can be approximated as monochromatic and of constant amplitude, and the system can be solved in steady state. The output, then, is at frequency  $\omega_1$ . In that case, the left-hand sides of (1) and (2) are 0, and (2) may be solved for the maximum frequency difference,



Fig. 3. Relationship between injection efficiency and maximum frequency offset demonstrates that, for a 100-cm perimeter cavity and injection-locked gain of 2, an injection efficiency of nearly 10% is required to lock two cavities, the natural frequencies of which vary by 10 MHz. The frequency offset in this figure is  $\nu = \omega/2\pi$ .

making use of the trigonometric requirement  $|\sin \varphi| \le 1$ 

$$|\omega_1 - \omega_0| \le \gamma_r \frac{E_1}{E_0} = \gamma_r \sqrt{\frac{I_1}{I_0}} \tag{3}$$

under the approximation  $E \approx E_0$ . The full locking bandwidth is just twice this number. Note that (3) does not depend on the actual laser system used, just the round-trip time of the cavity. The locking relationship is shown graphically in Fig. 3.

#### B. Regenerative Gain and Bandwidth

The injection-locked system can be described as a regenerative amplifier. To determine the gain of this amplifier, (1) and (2) are squared in steady state and added. Making the approximation  $-\ln(R_O) \approx 1 - R_O$ , and using the definition of  $\gamma_e$ , the gain can be calculated [18]

$$G(\omega) \approx \frac{\eta}{1 - R_O} \frac{4\gamma_e^2}{(\gamma_c - \gamma_m)^2 + 4(\omega_1 - \omega_0)^2}$$
$$= \frac{\eta}{1 - R_O} \left(\frac{2\gamma_e}{\gamma_c - \gamma_m}\right)^2 \frac{1}{1 + \left(2\frac{\omega_1 - \omega_0}{\gamma_c - \gamma_m}\right)^2}$$
$$= G_0 \frac{1}{1 + \left(\frac{\omega_1 - \omega_0}{\Delta\omega/2}\right)^2}$$
(4)

where

$$G_0 \equiv \frac{\eta}{1 - R_O} \left(\frac{2\gamma_e}{\gamma_c - \gamma_m}\right)^2 \tag{5}$$

is the standard on-resonance gain of a cavity and

$$\Delta \omega \equiv \gamma_c - \gamma_m \tag{6}$$

is the full-width at half-maximum (FWHM) bandwidth of the regenerative amplifier gain. In an injection-locked laser,  $\gamma_c$  is very slightly larger than  $\gamma_m$  (in a free-running laser they are equal). Using (1) in steady state, (6) becomes

$$\Delta\omega < 2\gamma_r \sqrt{\frac{I_1}{I_0}} \tag{7}$$

so the regenerative-gain bandwidth equals the injection-locking bandwidth.

#### C. Injection-Locked Laser Output

The expected output from the injection-locked laser at  $\omega_1$  is calculated from (1), again making the steady-state assumption. In homogeneous media, as has been assumed for this paper, the saturated gain is related to the small-signal gain through

$$g(I) = \frac{g_0}{1 + I/I_{\text{sat}}} \tag{8}$$

where  $I_{\text{sat}}$  only depends on the laser frequency  $\omega$  and constant material parameters. With this gain profile, defining the ratio of pump rate to threshold pump rate as  $r \equiv g_0/g$ , an equation for injected laser output intensity can be derived [18]:

$$\frac{(r-1)(I-I_0)}{(r-1)I+I_0} = 2\frac{\gamma_r}{\gamma_c}\sqrt{\frac{I_1}{I_0}}\cos\varphi$$
(9)

which is a third-order equation with a complicated closed-form solution. Making the standard injection-locking assumption that  $I \approx I_0$ , however, (9) can be solved for the output intensity

$$I_{\rm out} = I_{\rm out}^{\rm fr} + \frac{2r}{r-1} \frac{\gamma_e}{\gamma_c} \sqrt{\frac{I_1}{I_0}} \cos\varphi \tag{10}$$

which is slightly greater than the laser free-running output  $I_{\text{out}}^{\text{fr}} \equiv (1 - R_O)I_0$ . Within the locking range, the injection-locked output described by (10) is shown in Fig. 4.

#### IV. MUTUAL INJECTION-LOCKING THEORY

## A. Locking Range

The above analysis can be extended to the case of mutual injection locking. There are three cavities to consider: Cavity 1 and Cavity 2, which are the independent oscillator cavities, and Cavity 3, the cavity including both gain media and the mutual injection (Fig. 5). Then the three intensity gains, calculated from regenerative amplifier equations in the small-signal region, are [18]

$$|g_1(\omega)|^2 \approx \frac{\gamma_1^2}{(\omega - \omega_1)^2}$$
$$|g_2(\omega)|^2 \approx \frac{\gamma_2^2}{(\omega - \omega_2)^2}$$



Fig. 4. Relative output of the cavity used as an example in this section shows that, within the locked range of  $\pm 7$  MHz, the system produces greater output at  $\nu_1$  than the free-running laser would at  $\nu_0$ . Outside the locked range, the system acts as a free-running oscillator. There is also a transition range over which both  $\nu_1$  and  $\nu_0$  are produced.



Fig. 5. Mutual injection locking shares portions of the signal from each cavity with the other, resulting in linking three cavities: the two independent oscillators, Cavity 1 and Cavity 2, and the virtual cavity formed by the coupling, Cavity 3.

$$|g_3(\omega)|^2 \approx \frac{\gamma_3^2}{(\omega - \omega_3)^2} \tag{11}$$

where each free-running frequency  $\omega_i$  and each round-trip rate  $\gamma_i$  are determined by its cavity perimeter  $p_i$ . Note that, because  $p_3$  is larger than  $p_1$  or  $p_2, \omega_3$  is always between  $\omega_1$  and  $\omega_2$ . Thus, this part of the denominator of  $g_3$  is usually smaller than the same part of the denominator of either  $g_1$  or  $g_2$ . On the other hand,  $\gamma_3 < \gamma_1, \gamma_2$ , so the gain  $|g_3(\omega)|^2$  is usually less than the other two intensity gains. The free-running intensity  $I_3(\omega_3)$ , however, is likely to be greater than the other free-running intensities, because it passes through both gain media, although it ordinarily has higher loss (because  $I_3$  only exists through the mutual injection, it is a factor of  $\eta_1\eta_2$  lower than it would be in its own cavity). As a general rule, then, the oscillators will lock at  $\omega_3$ . This will happen as long as both the following are true:

$$I_{3}(\omega_{3}) > \frac{\eta_{1}^{2}\eta_{2}}{(\omega_{1} - \omega_{3})^{2}\tau_{3}^{2}}I_{1}(\omega_{1})$$

$$I_{3}(\omega_{3}) > \frac{\eta_{1}\eta_{2}^{2}}{(\omega_{2} - \omega_{3})^{2}\tau_{2}^{2}}I_{2}(\omega_{2}).$$
(12)



Fig. 6. Level of intracavity coupling required for full mutual injection locking increases with  $\beta$ , the coupling parameter defined in (14).

If neither is true, it is possible for the two lasers to lock at either  $\omega_1$  or  $\omega_2$ , as long as the injected intensity at that frequency is greater than any of the other intensities, whereas if one is true but not the other, the lasers are unlikely to lock together. Increasing the pump forces locking at  $\omega_3$ , because the gain seen by  $I_3$  is the product of the gains seen by  $I_1$  and  $I_2$ . Increasing the pump, then, increases the gain of  $I_3$  more than the gain of either other beam.

In the case of symmetric injection locking, where  $\eta_1 = \eta_2 \equiv \eta$ , (12) are also symmetric. One of the two independent freerunning intensities will be larger than the other; without loss of generality, we select  $I_1 > I_2$ . Then the cavities will lock at  $\omega_3$ as long as

$$\frac{I_3}{I_1} > \frac{\eta^3}{(\omega_1 - \omega_3)^2 \tau_3^2}.$$
(13)

Equation (13) can be interpreted as a restriction on the injection efficiency,  $\eta$ . Because  $I_3$ , in passing through the two gain media, is reinjected twice per round trip with gain in both media,  $I_3$  is proportional to  $\eta^4(\omega_1 - \omega_3)^2\tau_1^2(\omega_2 - \omega_3)^2\tau_2^2$ . Therefore, (13) can be rewritten

$$\eta > \beta \frac{\tau_1^2}{\tau_3^2} (\omega_2 - \omega_3)^2 \tau_2^2 \tag{14}$$

where the coupling parameter

$$\beta \equiv \frac{I_1}{I_3} \eta^4 \tag{15}$$

is independent of  $\eta$ . The coupling parameter  $\beta$  is usually small. The other factors in (14) include  $\tau_2^2$ , which is small, and  $\tau_1^2/\tau_3^2$ , which is <1, so  $\eta$  can also be very small. A plot of  $\eta$  required for a sample two-cavity system with 2-ns round-trip time in each and 5 ns for the mutual cavity is shown for sample values of  $\beta$  in Fig. 6. As this figure shows, the locking bandwidth for mutual injection locking is significantly larger than that for traditional injection locking. Indeed, because the mode spacing of the injection cavity is 200 MHz in this example, the total required injection efficiency is likely to be <10%, even for high values of  $\beta$ . Because the coupling requirements come from phase values, the effective frequency offset is calculated modulo the mode separation frequency.

# B. Phase Shift Between Adjacent Oscillators

Regenerative amplification causes a phase shift, as demonstrated in Section III. In the case of mutual injection locking, this results in a phase shift between adjacent oscillators. The absolute phase shift is not critical. Maximum brightness, however, depends on the phase shift between oscillators *at their output*. In other words, if the difference between the phase of output from Cavity 2 *at the output of Cavity 1* and the Cavity 1 phase at the same point is a multiple of  $2\pi$ , the lasers will add constructively, resulting in the highest possible on-axis intensity at any distance. This phase is a function of spacing between the cavities, positions of the output mirrors, and optical cavity perimeter *p*. To maintain optimal coherent addition one of these three must be adjustable.

To calculate the phase difference we return to the SVEA equations. Rewriting (1) as two coupled equations

$$\frac{dE_1}{dt} + \frac{\gamma_{c1} - \gamma_{m1}}{2} E_1 = \gamma_1 \eta_2 E_2 \cos(\varphi_1 - \varphi_2)$$
$$\frac{dE_2}{dt} + \frac{\gamma_{c2} - \gamma_{m2}}{2} E_2 = \gamma_2 \eta_1 E_1 \cos(\varphi_1 - \varphi_2) \quad (16)$$

where the numbered subscripts correspond to the cavities and the cosine term does not change because  $\cos(\varphi) = \cos(-\varphi)$ . The injection efficiency from Cavity 1 into Cavity 2 is  $\eta_1$  and that from Cavity 2 into Cavity 1 is  $\eta_2$ . At steady state, (16) has the solution

$$\begin{aligned} \varphi_1 - \varphi_2 | &= \cos^{-1} \left( \sqrt{\frac{\gamma_{c1} - \gamma_{m1}}{2\gamma_1 \eta_1}} \frac{\gamma_{c2} - \gamma_{m2}}{2\gamma_2 \eta_2} \right) \\ &\approx \frac{\pi}{2} - \sqrt{\frac{\gamma_{c1} - \gamma_{m1}}{2\gamma_1 \eta_1}} \frac{\gamma_{c2} - \gamma_{m2}}{2\gamma_2 \eta_2} \end{aligned} \tag{17}$$

for the typical case of  $\gamma_c - \gamma_m = \gamma_r$  for each cavity. Thus, the phase difference between adjacent oscillators *at their own outputs* is approximately 90°.

# C. Power Output

The output of the two oscillators is calculated from (2), which is rewritten for the coupled oscillators as

$$\frac{d\varphi_1}{dt} + (\omega_1 - \omega_3)E_1 = \gamma_1\eta_2 E_3 \sin\varphi_1$$
$$\frac{d\varphi_2}{dt} + (\omega_2 - \omega_3)E_2 = \gamma_2\eta_1 E_3 \sin\varphi_2.$$
(18)

From (17) we know

$$\sin(\varphi_1 - \varphi_2) = \sqrt{1 - \frac{\gamma_{c1} - \gamma_{m1}}{2\gamma_1 \eta_1} \frac{\gamma_{c2} - \gamma_{m2}}{2\gamma_2 \eta_2}}.$$
 (19)

Likewise, we know that the ratio of the phases will equal the ratio of the frequency offsets from  $\omega_3$ . Choosing the measuring location so that  $\varphi_3 = 0$ , (18) becomes, under steady-state

conditions,

$$E_1 = \frac{\gamma_1 \eta_2}{\omega_1 - \omega_2} E_3 \sin(\varphi_1 - \varphi_2)$$
$$E_2 = \frac{\gamma_2 \eta_1}{\omega_1 - \omega_2} E_3 \sin(\varphi_1 - \varphi_2). \tag{20}$$

In a symmetric cavity, where  $\eta_1 = \eta_2 = \eta$ , (20) combines with (19) to reveal the intensity equation inside the cavities

$$I_1 \approx \left(\frac{2\eta\gamma_1}{\omega_1 - \omega_2}\right)^2 \frac{\gamma_1}{\gamma_2} I_3$$
$$I_2 \approx \left(\frac{2\eta\gamma_2}{\omega_1 - \omega_2}\right)^2 \frac{\gamma_2}{\gamma_1} I_3.$$
(21)

The signal intensity in each cavity, then, is greater than the free-running  $I_3$ , which itself is greater than the free-running intensity of both independent cavities. The cavity with the shorter round-trip time will have the higher output. The output intensity of each is just the output coupler transmission multiplied by the cavity intensity. To this level of approximation, the output from each cavity is proportional to the transmission of the output coupler and the round-trip time of the *other* cavity and is inversely proportional to the cube of the round-trip time of its own cavity. The output, to this approximation, is independent of the loss or gain in the cavity.

#### V. EXPERIMENT

#### A. Layout

The system used for this test is shown in Fig. 7. A Coherent Verdi laser system capable of up to 8 W at 532 nm was used as the pump source. Its beam was separated into two equal intensities by a beamsplitter. Each of these two beams passed through a lens and the high reflector of the cavity before reaching its focal point within the laser crystal. Each cavity also contained an output coupler and a pickoff mirror for coupling.

The pump beams were polarized along the *c*-axis of the two Nd:YVO<sub>4</sub> laser crystals. Laser emission at 1.064  $\mu$ m was polarized along the *a*-axis. The crystals were from different boules and of different geometry; one was  $3 \times 3 \times 10$  mm with faces coated for low reflectivity at 1.064  $\mu$ m, and the other was  $2 \times 3 \times 13.1$  mm with uncoated faces. The cavities were both 23 cm long and contained identical mirrors. The high reflectors were 99.2% reflective, and the output coupler reflectivities were 95.8% at 1.064  $\mu$ m. The high reflectors were spherical, with a radius of 3 m, whereas the output couplers were flat. The calculated lowest order mode size was ~400  $\mu$ m. Cavity 1, containing the laser crystal with coated surfaces, was pumped with a 600- $\mu$ m beam.

The two cavities also contained coupling mirrors. In each cavity was a partial reflector, oriented at  $45^{\circ}$ , with reflectivity 16.4% at 1.064  $\mu$ m for this angle of incidence. These mirrors were aligned so that 16.4% of the oscillating intensity in each cavity was directed to the coupling mirror in the other cavity, with the beams exactly counterpropagating. Because all angles of incidence were  $45^{\circ}$ , insertion of the pickoff mirrors into



Fig. 7. Mutual injection locking experiment included two independent laser oscillators, coupling within the cavities, and a method of combining the two output beams to measure mutual coherence.



Fig. 8. By plotting oscillation threshold versus mirror reflectivity, one can extrapolate the internal cavity losses and demonstrate the effects of pump size, shown by the different slopes of the two lines.

the cavities resulted in a round-trip loss of nearly 70% with a maximum coupling of 2.69%. Between these two mirrors was a polarizer that could be rotated to adjust the intensity of the throughput. The polarizer had the effect of multiplying the intensity passing through it by  $\cos^2 \theta$ , where  $\theta$  is the rotation angle, because the actual intensity was reduced to  $\cos \theta$  and the polarization was rotated to  $\theta$  away from the oscillating signal.

## B. Measurements of Independent Oscillation

Each oscillator was tested independently, with the coupling path blocked, to determine its properties. From the cavity length the round-trip time was calculated as 1.43 ns, for a mode spacing of  $\gamma_r = 700$  MHz. Laser output of each cavity was measured to determine oscillation threshold. This was performed for output couplers of various reflectivities, all flat. The plot of threshold versus mirror reflectivity is shown in Fig. 8.

Fig. 8 demonstrates that the internal single-pass loss of each cavity was  $\sim 19\%$ , so the round-trip loss was 34%. Unlike most

lasers, this system has internal losses significantly greater than the output coupling. The mode of each independent laser was also measured (Fig. 9). As expected from the mismatch between pump beam diameter and lowest order mode diameter, the beam quality from Cavity 1 is significantly worse than that from Cavity 2.

# C. Operation During Mutual Injection

The lasers were operated with the coupling in place, but with the 10000:1 polarizer set to block injection. The oscillation threshold—defined for this experiment as the pump power necessary to produce  $50-\mu$ W output—was 1.9 W for Cavity 2 and 6.3 W for Cavity 1, using the 95.8% R output couplers. The first effects of injection locking, a lowering of the lasing threshold for both cavities, were seen when mutual injection was increased to 0.03%. As coupling was increased, the threshold dropped even more (see Table II) until, at 0.5% mutual injection, the threshold was at its lowest. The threshold did not decrease with further coupling increases. The mode quality improved as coupling was increased past 0.03%, especially in Cavity 1. Finally, at 0.5% coupling, the modes of both lasers were good, and the lasers could be considered fully injection-locked.

The data in Table II demonstrate that mutual injection coupling is a good method of lowering the threshold of lasers. It is also clear that most of the threshold-lowering effect appears with less coupling than that needed to lock the lasers (0.5%). It is interesting to note that even 0.03% coupling is enough to reduce the lasing threshold in Cavity 1 to below its independent threshold by nearly a factor of 3. This corresponds well to (21). A plot of required injection efficiency as a function of coupling parameter is shown in Fig. 10. Estimating  $\beta$  in this experiment as 0.05, the required injection efficiency of 0.5% corresponds to a frequency offset ~250 MHz. The mode spacing of Cavity 3 is ~400 MHz in this experiment, so the estimated frequency offset is reasonable.



Fig. 9. Beam quality observed from (a) Cavity 1 is significantly lower than that from (b) Cavity 2.

 TABLE II

 Relation Between Injection and Oscillation Threshold Reduction

Threshold	Cavity 1	Cavity 2
Injection Efficiency = 0	6.3 W	1.9 W
Injection Efficiency = 0.03%	2.2 W	1.8 W
Injection Efficiency = $0.1\%$	1.9 W	1.5 W
Injection Efficiency = $0.5\%$	1.8 W	1.4 W
Injection Efficiency = 2.7%	1.8 W	1.4 W



Fig. 10. As cavity parameter and frequency offset increase, the injection efficiency required for mutual injection locking also increases.



Fig. 11. (a) At mutual injection efficiency 0.4%, even Cavity 1 is oscillating in low-order modes. (b) At 0.5% efficiency, the pattern formed by interference between outputs of the two cavities demonstrates that they are fully locked, operating at the same frequency with only a constant phase difference.

When the two laser cavities are fully injection-locked, their phase is locked to some small (and constant) difference. This can be seen from Fig. 11, which shows the results of mixing the two beams together. As injection percentage increases, laser threshold drops (Table II), and beam quality increases [compare Figs. 9(a) and 11(a)]. At approximately 0.5% injection in this experiment, the threshold and beam quality effects have saturated, and the two laser outputs are locked in phase and wavelength. No further improvement in threshold, output power, or beam quality is seen by increasing coupling from 0.5% to 2.7%.

## VI. SUMMARY

Mutual injection locking of lasers is a promising technology for the next generation of high-power laser arrays. This technique reduces laser threshold in the associated cavities, increases the lockable difference between the free-running laser wavelengths of the lasers, and reduces the phase difference between the locked oscillators. An experiment demonstrated that the laser thresholds of two independent Nd:YVO<sub>4</sub> laser cavities, pumped by a single 532-nm laser, were reduced up to a factor of 3.5, with significant mode quality improvement when the two cavities were mutually injection-locked. In this experiment, 0.5% mutual injection locking was required to fully phase-lock the two cavities, demonstrating the advantages of mutual injection locking over the traditional method. This injection level corresponds to a frequency difference of 5 MHz for master–slave injection locking, but 250 MHz for mutual injection locking. The relationship between frequency separation of the injected and free-running signals to the required injection power puts severe limits on the capabilities of master–slave injection-locked lasers. These limits are overcome by mutual injection locking.

#### REFERENCES

- H. T. Powell and H. L. Chen, Laser Science and Technology Program Update. Livermore, CA: Lawrence Livermore National Laboratories, Dec. 2000.
- [2] H. A. Jones-Bey, "Livermore laser targets battlefield environment," *Laser Focus World*, vol. 39, no. 12, pp. 42–45, Dec. 2003.
- [3] J. P. Machan, W. H. Long Jr., J. Zamel, D. Stucker, and L. Marabella, "5 kW diode-pumped solid state laser," presented at the Solid State and Diode Laser Tech. Rev., Albuquerque, NM, 2002.
- [4] G. Goodno, S. Palese, W. Long, J. Harkenrider, W. Burt, and H. Injeyan, "Evolution of high brightness solid-state lasers at TRW," presented at the Optical Society of America Annu. Conf., Long Beach, CA, 2001.
- [5] P. K. Cheo, A. Liu, and G. G. King, "A high-brightness laser beam from a phase-locked multicore Yb-doped fiber laser array," *IEEE Photon. Technol. Lett.*, vol. 13, no. 5, pp. 439–441, May 2001.
- [6] T. Y. Fan et al., "Laser beam combining for power and brightness scaling," presented at the Aerospace Conf., Big Sky, MT, 2000.
- [7] M. Musha, T. Kanaya, K. Nakagawa, and K. Ueda, "Coherent addition of two injection-locked Nd:YAG lasers," presented at the CLEO/Pacific Rim, Chiba, Japan, 2001.
- [8] E. A. Cummings, M. S. Hicken, and S. D. Bergeson, "Demonstration of a 1-W injection-locked continuous-wave Titanium:Sapphire laser," *Appl. Opt.*, vol. 41, no. 36, pp. 7583–7587, Dec. 2002.
- [9] W. W. Chow, "Theory of line narrowing and frequency selection in an injection locked laser," *IEEE J. Quantum Electron.*, vol. 19, no. 2, pp. 243– 249, Feb. 2003.
- [10] G.-R. Lin, "Mutual injection locking of an erbium-doped fiber laser and a fiber-pigtailed fabry-perot laser diode," *Opt. Lett.*, vol. 28, no. 14, pp. 1203–1205, Jul. 2003.
- [11] M. M. Ibrahim, "On injection locking of homogeneously broadened lasers," *IEEE J. Quantum Electron.*, vol. 14, no. 3, pp. 145–147, Mar. 1978.
- [12] Q. Wang, L. Yan, P.-T. Ho, G. Wood, M. Dubinskiy, and B. Zandi, "Mutual injection locking of two individual Nd:YVO<sub>4</sub> lasers in a compound cavity," presented at the Annual Conf. Lasers and Electro-Optics Soc., Tucson, AZ, 2003.
- [13] R. Kurtz, "Maritime advanced ship-defense thermally-managed intensely focused laser," US Navy—NavSea, Final Report N00178-03-C-3052, Jun. 2003.
- [14] H. L. Stover and W. H. Steier, "Locking of laser oscillators by light injection," *Appl. Phys. Lett.*, vol. 8, no. 4, pp. 91–93, Feb. 1966.
- [15] V. Annovazzilodi and S. Donatoi, "Injection modulation in coupled laser oscillations," *IEEE J. Quantum Electron.*, vol. 16, no. 8, pp. 859–864, Aug. 1980.
- [16] J. K. Butler, D. E. Ackley, and D. Botez, "Coupled-mode analysis of phase-locked injection laser arrays," *Appl. Phys. Lett.*, vol. 44, no. 3, pp. 293–295, Feb. 1984.
- [17] A. E. Siegman, *Lasers*. Mill Valley, CA: University Science, 1986.
- [18] R. M. Kurtz, "Derivations from paper," [Online]. Available: http://www. rmkurtz.com/injlock/index.html



**Russell M. Kurtz** (S'85–M'01–SM'01) received the S.B. degree in electrical engineering from the Massachusetts Institute of Technology, Cambridge, in 1981 and the Ph.D. degree in quantum electronics and optics from th1e University of Southern California, Los Angeles, in 1991.

He worked as an Engineer at General Electric, Hughes Aircraft, TRW, and the Aerospace Corporation Howthorne, CA and as Consulting Engineer to Allied-Signal, LIWA, and Beagle I before joining Physical Optics Corporation (POC), Torrance, CA as

a Team Leader. While at POC he has worked on a wide variety of research topics,

including 3-D displays, scene generators, methods of nondestructive evaluation, nanotechnology, measurement of nanomotion, and advanced laser systems and was promoted to Group Leader in 2004. He has recently presented results of this research at the SPIE Defense and Security Symposium and at CLEO.

Dr. Kurtz is a Member of the Optical Society of America, SPIE, AAAS, ASNT, and the Directed Energy Professional Society, as well as IEEE-LEOS. He is the immediate Past President of the MIT Club of Southern California.

**Ranjit D. Pradhan** received the B.Sc. degree in physics from Bombay University, Bombay, India, in 1987, and the M.Sc. degree in physics and the M.Tech. degree in materials science from the Indian Institutes of Technology in 1989 and 1991, respectively. He received the Ph.D. degree in physics from the University of Delaware, Newark, in 1997.

He worked as a Postdoctoral Research Fellow and later Research Associate at the Applied Optics Center (AOC) of Delaware at the University of Delaware, Newark, where he was instrumental in building research facilities for the nascent center. His current research covers miniature optical devices including spectral sensors, tunable filters, and waveguide-based devices. In addition he is involved in the development of nondestructive testing instruments, infrared reflectometers, and 3-D and head-mounted displays.

Dr. Pradhan is a Member of the Optical Society of America and has served as a referee for *Physical Review Letters* and *Optics Letters*.

**Nay Tun** received the B.S. degree in electronic engineering from the Rangoon Institute of Technology, Rangoon, Burma, in 1998.

Before joining Physical Optics Corporation (POC), Torrance, CA, he was an Electronics Engineer with the Shanpia International R&D Department, where he designed and developed circuitry for automotive accessories. After serving in the U.S. Army for four years, he joined California Amplifier, Inc. as an Associate Engineer. There he was a Test Equipment Engineer and made significant contributions to prototyping communication equipment. At POC he has developed driver electronics to precisely scan and control a collimated beam in a U.S. Army smart munition. He has designed and fabricated electronic drivers and control systems for various LCD-based devices.

**Tin M. Aye** received the Ph.D. degree in theoretical physics (optics) from the Imperial College of Science and Technology, London, U.K., in 1983.

Since coming to Physical Optics Corporation (POC), Torrance, CA, in 1988, he has developed holographic technologies for a broad range of optoelectronic devices and display systems. Currently, he is directing the development of holographic and diffractive optic devices for beam scanning, polarization filtering, microdisplays, head-mounted displays, and collimating screens for simulators. He was instrumental in the breakthrough development of light-shaping diffusers, POC's main product line. His accomplishments include the development of coherently coupled compound holographic filters for coherence discrimination, volume holographic diffusers, a tunable holographic Fabry-Perot etalon, a holographic submicron lithography system, and high-efficiency large area holographic optical elements for laser radar, imaging, display, and security applications. In earlier positions with General Optics Technologies, Inc. and Teledyne Solid State Products. He was responsible for developing holographic technology for information display, security, communications, and optical coupling in microelectronics.

**Gajendra D. Savant** received the Ph.D. degree in polymer chemistry from Shivaji University, Kolhapur, India, and the National Chemical Laboratory, Pune, India, where he synthesized a series of high-performance polymers for thermal, optical, and aerospace applications.

Under a Postdoctoral Fellowship, he continued his research at the California Institute of Technology and New Mexico Tech. Currently, he is involved in the development of 3-D stereoscopic displays, spatial light modulators, microlithographic systems for flat-panel displays, multispectral imaging, and nondestructive evaluation systems. Recently, he developed an efficient day-lighting system based on holographic technology and an optical disk for high-density data storage. In addition, he developed proprietary materials for production of holographic light-shaping diffusers, display screens, seamless manufacturing of backlight films, and a fiber-optic connector to be produced by plastic molding. For the past several years, he has been actively involved in the commercialization and manufacturing of diffusers, 3-D displays, backlights, and wavelength division multiplexers. He is the inventor of a new class of polymer-based grafts for extreme ultraviolet holography.

**Tomasz P. Jannson** received the B.S. degree in electrical engineering and the Ph.D. degree in applied optics from Warsaw Technical University, Warsaw, Poland, in 1965 and 1973, respectively.

He worked as a Senior Research Scientist at Northrop Corporation Howthorne, CA and a Chief Scientist at National Technical Systems Torrance, CA, before helping create Physical Optics Corporation (POC), in 1984 and becoming its Chief Scientist. While at POC he has been in charge of all commercial technologies. In association with Prof. Emil Wolf of the University of Rochester, he developed radiometric ray-tracing technology. He was also fundamental to the development of autostereoscopic 3-D imaging, non-Lambertian diffuser technology, fiber-optic multimedia communications, and fiber-optic sensors based on wavelength division multiplexing.

Dr. Jannson is a Member of OSA and a Fellow of SPIE. He has been the Chair or Co-Chair of several SPIE conferences, including a special conference on the life work of Emil Wolf.



**Larry G. DeShazer** (F'94) received the Ph.D. degree in physics (atomic spectroscopy) from The Johns Hopkins University, Baltimore, MD, in 1963.

He has more than 40 years of experience in solidstate physics, solid-state lasers, optical materials, and electro-optical devices. He had tenure at the University of Southern California (USC), Los Angeles, in the departments of physics and electrical engineering as a Teaching and Research Professor of laser physics and quantum electronics, serving for 13 years. He founded the Center for Laser Studies at USC, an ap-

plied research organization interfacing between the university and industry. He was Director of the Laser Systems Division, McDonnell Douglas Aerospace, St. Louis, MO; Head of the Laser Optical Materials Section, Hughes Research Laboratories, Malibu, CA; and Director of the Solid State Laser Group, Spectra Technology, Inc., St. Louis, MO, a subsidiary of Spectra Physics. He was President of Solidlite Corporation in Redmond, WA; BioLase Technology, Inc., in San Clemente, CA; and Future Light LLC in Tustin, CA, all of which made laser medical products. He was Liaison Scientist for the U.S. Office of Naval Research in London during 1975–1976, assisting U.S. and European scientists in establishing contacts. He was also Visiting Professor at Rennes University in France and Visiting Scientist to the Academy of Science USSR. Currently he is Director of the Center for Applied Competitive Technologies (CACT), a California State Economic and Workforce Initiative located at Irvine Valley College, Irvine, CA, and is Adjutant Professor of Physical Science at the College.

Dr. DeShazer is a Fellow of the Optical Society of America, the Treasurer of the Optics Institute of Southern California, and a Member of Sigma Xi, and SPIE. He was Associate Editor of the IEEE JOURNAL OF QUANTUM ELECTRONICS from 1985–1991.